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Amplitude reduction of parametric resonance by Velocity Feedback control

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Abstract
The response of a dynamical non-linear system of two-degree-of freedom, is investigated. Analysis of the amplitude and phase plane is obtained. The method of multiple time scale is applied to solve the non-linear differential equations describing the system up to second order approximation. All possible resonance cases at this approximation are obtained and studied numerically to determine the worst case. The effects of different parameters are studied. The frequency response equations are solved numerically. These vibrations were controlled using the damper $(R_1 = -\varepsilon G_1 \dot{y}^3,\ R_2 = -\varepsilon G_2 \dot{\theta}^3)$

Keywords: Resonance; vibration control; Frequency response function; Multiple time Scale.

تقليل سعة الرنين البارامتري عن طريق التحكم في السرعة التراجعية

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الملخص

يتم التقصي على استجابة نظام ديناميكي غير خطي من الدرجة الثانية. بحيث يتم الحصول على السعة تحليليا. ويتم تطبيق طريقة المقياس الزمني المتعدد لحل المعادلات التفاضلية غير الخطية التي تصف النظام الى التقريب من الدرجة الثانية. يتم الحصول على جميع حالات الرنين الممكنة عند هذا التقريب ودراستها عدديا لتحديد الحالة الاسوأ.

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وتمت دراسة تأثير المعاملات المختلفة. ويتم حل معادلات الاستجابة عدديا. يتم التحكم في هذه الاهتزازات باستخدام المخمد $(R_1 = -\varepsilon G_1 \dot{y}^3, R_2 = -\varepsilon G_2 \dot{\theta}^3)$.

الكلمات الدالة: الرنين. التحكم في الاهتزاز وظيفة استجابة التردد؛ مقياس زمني متعدد.

1. Introduction

Principal parametric resonance occurs in systems having time-varying coefficients through pitchfork bifurcation in the case when the excitation frequency is in the neighborhood of twice the natural frequency of the system. The book written by Nayfeh and Mook [1] showed the mechanism of parametric resonance and history of research concerning this one, i. e., Faraday who has been the first to recognize the phenomenon of parametric resonance and Melde who performed the first serious experiments on parametric resonance. From physical and engineering points of view, the parametric resonance has been attractive, and many researches on the phenomena and their utilization have been continuously performed until now. Pratiher et al. [2] investigated parametric instabilities of a cantilever beam with magnetic field and axial load. Huang and Kuang [3] studied the parametric resonance instabilities in a drilling process. Shaw and Baskaran [4] investigated the use of parametric resonance to improve filtering characteristics in microelectromechanical filters. Piccardo and Tubino [5] analyzed the excessive lateral sway motion caused by crowds walking across footbridges using parametric excitation mechanism. Racz and Scott [6] investigated the parametric

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instability in a finite-length rotating cylinder subjected to periodic axial compression by small sinusoidal oscillations of the piston. Sayed and Hamed [7] studied the response of a two degree-of freedom system with quadratic coupling under parametric and harmonic excitations. The method of multiple scale perturbation technique is applied to solve the non-linear differential equations and obtain approximate solutions up to and including the second-order approximations.

In the present paper, the non-linear vibrations and stability subjected to the transverse and in-plane excitations simultaneously are investigated. The method of multiple time scale is applied to obtain the second-order uniform asymptotic solutions. All possible resonance cases are extracted and investigated at this approximation order. It is quite clear that some of the simultaneous resonance cases are undesirable in the design of such system. Such cases should be avoided as working conditions for the system. The stability of the is investigated with frequency response curves and phase-plane method. Some recommendations regarding the different parameters of the system are reported.

2. Mathematical Analysis

The system was previously studied without control

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$$\ddot{y}+2\varepsilon\mu_1\dot{y}+\left(1+2\varepsilon a_e k_2\cos\Omega t\right)y+\varepsilon\alpha_2 y^2+\varepsilon\alpha_3 y^3=-\varepsilon c_1(\mathrm{y}\,\dot{y}^2+y^2\ddot{y})$$
$$-\varepsilon c_2\ddot{\theta}y+\varepsilon a_e k_1\cos\Omega t-\varepsilon^2 a_e{}^2 k_2\cos^2\Omega t+\varepsilon^3 a_e{}^3 k_3\cos^3\Omega t-3\varepsilon^2 a_e{}^2 k_3\,\mathrm{y}$$
$$\cos^2\Omega t+3\varepsilon a_e k_3\cos\Omega t y^2$$

$$\ddot{\theta}+2\varepsilon\mu_2\dot{\theta}+\omega_\theta^2\theta=-\varepsilon c_3 y\ddot{y}-\varepsilon c_3\dot{y}^2+\varepsilon^2\theta c_4 a_e\cos\Omega t-\frac{1}{6}\varepsilon\omega_\theta^2\theta^3$$

In this section, we will study the system with control

$$\ddot{y}+2\varepsilon\mu_1\dot{y}+\left(1+2\varepsilon a_e k_2\cos\Omega t\right)y+\varepsilon\alpha_2 y^2+\varepsilon\alpha_3 y^3=-\varepsilon c_1(\mathrm{y}\,\dot{y}^2+y^2\ddot{y})$$
$$-\varepsilon c_2\ddot{\theta}y+\varepsilon a_e k_1\cos\Omega t-\varepsilon^2 a_e{}^2 k_2\cos^2\Omega t+\varepsilon^3 a_e{}^3 k_3\cos^3\Omega t-3\varepsilon^2 a_e{}^2 k_3\,\mathrm{y}$$
$$\cos^2\Omega t+3\varepsilon a_e k_3\cos\Omega t y^2+R_1 \quad (1)$$

$$\ddot{\theta}+2\varepsilon\mu_2\dot{\theta}+\omega_\theta^2\theta=-\varepsilon c_3 y\ddot{y}-\varepsilon c_3\dot{y}^2+\varepsilon^2\theta c_4 a_e\cos\Omega t-\frac{1}{6}\varepsilon\omega_\theta^2\theta^3+R_2$$
$$(2)$$

where,
$R_1=-\varepsilon G_1\dot{y}^3$, $R_2=-\varepsilon G_2\dot{\theta}^3$, and y,θ
are the vibration amplitudes of the composite laminated rectangular thin plate for the first-order and the second-order modes, respectively, μ_1 and μ_2 the damping coefficients, ω_θ the linear natural frequency of the thin Plate, and Ω the excitation frequency, α_2 and α_3 are the coefficients of Taylor expansion of the magnetic force with respect to y^2 and y^3 respectively, c_1,c_2,c_3 and c_4 are the coefficients determined by system parameters. The coefficients of linear, quadratic, and cubic terms by Taylor expansion of force with respect to y are k_1,k_2 and k_3, respectively. Also, G_1,G_2 are gain coefficients.

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We seek a second order uniform expansion for the solutions of equation (1) in the form:

$$y(t,\varepsilon)=y_0(T_0,T_1)+\varepsilon y_1(T_0,T_1)+\varepsilon^2 y_2(T_0,T_1)+O(\varepsilon^3) \qquad (3)$$

$$\theta(t,\varepsilon)=\theta_0(T_0,T_1)+\varepsilon\theta_1(T_0,T_1)+\varepsilon^2\theta_2(T_0,T_1)+O(\varepsilon^3) \qquad (4)$$

where $T_n=\varepsilon^n t,\ (n=0,1,2)$, and the time derivatives became

$$\frac{d}{dt}=D_0+\varepsilon D_1+\dots\ , \qquad \frac{d^2}{dt^2}=D_0^2+2\varepsilon D_0 D_1+\varepsilon^2 D_0^2+\dots, \qquad (5)$$

where and ε is small a perturbation parameter and $0<\varepsilon<<1$, T_0 is the fast time scale, T_1 is the slow time scales.

Substituting equations (3), (4) and (5) into equations (1) and (2) and equating the coefficients of same power of ε in both sides, we obtain the following set of ordinary differential equations:

Order ε^0:

$$\left(D_0^2+1\right)y_0=0 \qquad (6)$$

$$\left(D_0^2+\omega_\theta^2\right)\theta_0=0 \qquad (7)$$

Order ε^1:

$$\begin{aligned}\left(D_0^2+1\right)y_1=&-2D_0D_1y_0-2\mu_1D_0y_0-2a_ek_2y_0\cos\Omega T_0-\alpha_2y_0^3-\alpha_3y_0^3\\&-c_1y_0(D_0y_0)^2-c_1y_0^2D_0^2y_0-c_2y_0D_0^2\theta_0+a_e\,\mathrm{k}_1\cos\Omega T_0\\&+3a_e\,\mathrm{k}_3\,y_0^2\cos\Omega T_0-G_1\left(D_0y_0\right)^3\end{aligned} \qquad (8)$$

$$\begin{aligned}\left(D_0^2+\omega_\theta^2\right)\theta_1=&-2D_0D_1\theta_0-2\mu_2D_0\theta_0-c_3y_0D_0^2y_0-c_3\left(D_0y_0\right)^2-\frac{1}{6}\omega_\theta^2\theta_0^3\\&-G_2\left(D_0\theta_0\right)^3\end{aligned} \qquad (9)$$

Order ε^2:

$$\begin{aligned}\left(D_0^2+1\right)y_2=&-2D_0D_1y_1-D_1^2y_0-2D_0D_2y_0-2\mu_1D_1y_0-2\mu_1D_0y_1\\&-2a_e\,\mathrm{k}_2\,y_1\cos\Omega T_0-2\alpha_2y_0y_1-3\alpha_3y_1y_0^2-2c_1y_0D_0y_0D_0y_1\\&-2c_1y_0D_0y_0D_1y_0-c_1y_1(D_0y_0)^2-c_1y_0^2D_0^2y_1-2c_1y_0^2D_0D_1y_0\\&-2c_1y_0y_1D_0^2y_0-c_2y_0D_0^2\theta_1-2c_2y_0D_0D_1\theta_0-c_2y_0D_0^2\theta_0-a_e^2k_2\end{aligned}$$

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$$\cos^2\Omega t - 3a_e^2 k_3 y_0 \cos^2\Omega t + 6a_e k_3 y_0 y_1 \cos\Omega t - 3G_1\left(D_0 y_0\right)^2 D_0 y_1$$
$$-3G_1\left(D_0 y_0\right)^2 D_1 y_0 \qquad (10)$$

$$\left(D_0^2+\omega_\theta^2\right)\theta_2 = -2D_0D_1\theta_1 - D_1^2\theta_0 - 2D_0D_2\theta_0 - 2\mu_2 D_1\theta_0 - 2\mu_2 D_0\theta_1$$
$$-2c_3 y_0 D_0 D_1 y_0 - c_3 y_0 D_0^2 y_1 - c_3 y_1 D_0^2 y_0 - 2c_3 D_0 y_0 D_0 y_1$$
$$-2c_3 D_0 y_0 D_1 y_0 + c_4 a_e \theta_0 \cos\Omega t - \frac{1}{2}\omega_\theta^2\theta_1\theta_0^2 - 3G_2\left(D_0\theta_0\right)^2 D_0\theta_1$$
$$-3G_2\left(D_0\theta_0\right)^2 D_1\theta_0 \qquad (11)$$

The general solution of equations (6) and (7) is given by

$$y_0(T_0,T_1) = A_0(T_1)\exp(iT_0) + \bar{A}_0(T_1)\exp(-iT_0) \qquad (12)$$

$$\theta_0(T_0,T_1) = B_0(T_1)\exp(i\,\omega_\theta T_0) + \bar{B}_0(T_1)\exp(-i\,\omega_\theta T_0) \qquad (13)$$

where A_0, B_0 are unknown functions in T_1 at this level of approximation and can be determined by elimination the secular terms from the next order of perturbation. Substituting equations (12) and (13) into equations (8), (9) yields

$$\left(D_0^2+1\right)y_1 = (-2iD_1A_0 - 2\mu_1 iA_0 - 3\alpha_3 A_0^2\bar{A}_0 + 2c_1A_0^2\bar{A}_0 - 3iA_0^2\bar{A}_0 G_1)$$
$$\exp(iT_0) - a_e k_2 A_0 \exp(i(1+\Omega)T_0) - a_e k_2 A_0 \exp(i(1-\Omega)T_0)$$
$$-\alpha_2 A_0^2 \exp(2iT_0) + (iG_1 - \alpha_3 + 2c_1)A_0^3\exp(3iT_0) + c_2\omega_\theta^2 A_0B_0$$
$$\exp(i(\omega_\theta+1)T_0) + c_2\omega_\theta^2 A_0\bar{B}_0\exp(i(1-\omega_\theta)T_0) + c_2\omega_\theta^2 A_0B_0$$
$$\exp(i(\omega_\theta+1)T_0) + c_2\omega_\theta^2 A_0\bar{B}_0\exp(i(1-\omega_\theta)T_0) + \frac{3}{2}a_e k_3 A_0^2$$
$$\exp(i(2+\Omega)T_0) + \frac{3}{2}a_e k_3 A_0^2\exp(i(2-\Omega)T_0) + (3a_e k_3 A_0\bar{A}_0$$
$$+\frac{a_e}{2}k_1)\exp(i\Omega T_0) - \alpha_2 A_0\bar{A}_0 + cc \qquad (14)$$

$$\left(D_0^2+\omega_\theta^2\right)\theta_1 = (-2i\,\omega_\theta D_1 B_0 - 2\mu_2 i\,\omega_\theta B_0 - \frac{1}{2}\omega_\theta^2 B_0^2\bar{B}_0 - 3i\,\omega_\theta^3 B_0^2\bar{B}_0 G_2)$$
$$\exp(i\,\omega_\theta T_0) + 2c_3A_0^2\exp(2iT_0) + (\frac{1}{6} + iG_2\,\omega_\theta)\omega_\theta^2 B_0^3$$
$$\exp(3i\,\omega_\theta T_0) + cc \qquad (15)$$

The general solutions of equations (14) and (15) are:

$$y_1(T_0,T_1) = A_1(T_1)\exp(iT_0) + E_1\exp(i(1+\Omega)T_0) + E_2\exp(i(1-\Omega)T_0)$$

$$+E_3\exp(2iT_0)+E_4\exp(3iT_0)+E_5\exp(i(\omega_\theta+1)T_0)+E_6\exp(i(1-\omega_\theta)T_0)$$
$$+E_7\exp(i(2+\Omega)T_0)+E_8\exp(i(2-\Omega)T_0)+E_9\exp(i\Omega T_0)+E_{10}+cc$$
(16)

$$\theta_1(T_0,T_1)=B_1(T_1)\exp(i\omega_\theta T_0)+E_{11}\exp(2iT_0)+E_{12}\exp(3iT_0)+cc$$
(17)

Substituting equations (12), (13), (16) and (17) into equations (10), (11) and solving the resulting equation, we get:

$$y_2(T_0,T_1)=A_2(T_1)\exp(iT_0)+E_{13}\exp(i(1+\Omega)T_0)+E_{14}\exp(i(1-\Omega)T_0)$$
$$+E_{15}\exp(2iT_0)+E_{16}\exp(3iT_0)+E_{17}\exp(i(\omega_\theta+1)T_0)+E_{18}\exp(i(1-\omega_\theta)T_0)$$
$$+E_{19}\exp(i(2+\Omega)T_0)+E_{20}\exp(i(2-\Omega)T_0)+E_{21}\exp(i\Omega T_0)+E_{22}\exp(i(1+2\Omega)T_0)$$
$$+E_{23}\exp(i(\omega_\theta+\Omega+1)T_0)+E_{24}\exp(i(\Omega+1-\omega_\theta)T_0)+E_{25}\exp(2i\Omega T_0)$$
$$+E_{26}\exp(i(3+\Omega)T_0)+E_{27}\exp(i(3-\Omega)T_0)+E_{28}\exp(2i(\Omega+1)T_0)+E_{29}$$
$$\exp(2i(1-\Omega)T_0)+E_{30}\exp(i(\omega_\theta+3)T_0)+E_{31}\exp(i(3-\omega_\theta)T_0)+E_{32}\exp(4iT_0)$$
$$+E_{33}\exp(5iT_0)+E_{34}\exp(i\omega_\theta T_0)+E_{35}\exp(i(4+\Omega)T_0)+E_{36}\exp(i(4-\Omega)T_0)$$
$$+E_{37}\exp(i(2-\Omega)T_0)+E_{38}\exp(i(\omega_\theta+2)T_0)+E_{39}\exp(i(2-\omega_\theta)T_0)+E_{40}$$
$$\exp(i(\omega_\theta+1-\Omega)T_0)+E_{41}\exp(i(1-\omega_\theta-\Omega)T_0)+E_{42}\exp(i(3\omega_\theta+1)T_0)$$
$$+E_{43}\exp(i(3\omega_\theta-1)T_0)+E_{44}\exp(i(\omega_\theta+\Omega+2)T_0)+E_{45}\exp(i(2\Omega+3)T_0)+E_{46}$$
$$\exp(i(3-2\Omega)T_0)+E_{47}\exp(i(\omega_\theta-\Omega+2)T_0)+E_{48}\exp(i(2-\omega_\theta-\Omega)T_0)$$
$$+E_{49}\exp(i(\omega_\theta+\Omega)T_0)+E_{50}\exp(i(\Omega-\omega_\theta)T_0)+E_{51}+cc$$
(18)

$$\theta_2(T_0,T_1)=B_2(T_1)\exp(i\omega_\theta T_0)+E_{52}\exp(2iT_0)+E_{53}\exp(3i\omega_\theta T_0)$$
$$+E_{54}\exp(3iT_0)+E_{55}\exp(4iT_0)+E_{56}\exp(iT_0)+E_{57}\exp(i(2+\Omega)T_0)$$
$$+E_{58}\exp(i(2-\Omega)T_0)+E_{59}\exp(i(1+\Omega)T_0)+E_{60}\exp(i(1-\Omega)T_0)+E_{61}$$
$$\exp(i\Omega T_0)+E_{62}\exp(i(2+\omega_\theta)T_0)+E_{63}\exp(i(2-\omega_\theta)T_0)+E_{64}$$
$$\exp(i(3+\Omega)T_0)+E_{65}\exp(i(3-\Omega)T_0)+E_{66}\exp(i(\omega_\theta+\Omega)T_0)$$
$$+E_{67}\exp(i(\omega_\theta-\Omega)T_0)+E_{68}\exp(i(2+2\omega_\theta)T_0)+E_{69}\exp(i(2-2\omega_\theta)T_0)$$
$$+E_{70}\exp(5i\omega_\theta T_0)+\mathrm{cc}\qquad(19)$$

where $E_n,(\mathrm{n}=1,\ldots,70)$ are complex functions in T_1 and cc denotes the complex conjugate terms.

From the above derived solutions, the reported resonance cases are:

$(i)\ \omega_\theta = \Omega = 0 \quad (ii)\ \omega_\theta \pm \Omega \cong 1 \quad (iii)\ \omega_\theta \cong \Omega - 1 \quad (iv)\ \omega_\theta - 2 \cong 0$
$(v)\ \omega_\theta = \pm\Omega \quad (vi)\ \omega_\theta \pm \Omega \cong 2 \quad (vii)\ \omega_\theta \cong \Omega - 2 \quad (viii)\ \Omega - 1 \cong 0$

3. Stability Analysis

We study the different resonance cases numerically to get the worst of them. One of the worst cases has been chosen to study the system stability. We introduce the detuning parameters σ_1 and σ_2 according to:

$$\Omega = 1 + \varepsilon\sigma_1,\ \omega_\theta = 2 + \varepsilon\sigma_2 \qquad (20)$$

Substituting equation (20) into equations (14) and (15) and eliminating the secular and small divisor terms from y_1 and θ_1, we get the following:

$$D_1A_0 + \mu_1 A_0 - \frac{3}{2} i\,\alpha_3 A_0^2\bar{A}_0 + c_1 i A_0^2\bar{A}_0 + \frac{3}{2} G_1 A_0^2\bar{A}_0 + \frac{c_2}{2} i\,\omega_\theta^2 \bar{A}_0 B_0 \exp(i\,\sigma_2 T_1)$$
$$+(\frac{a_e}{4}\mathrm{ik}_1 + \frac{3}{2}\mathrm{ia}_e\mathrm{k}_3 A_0\bar{A}_0)\exp(i\,\sigma_1 T_1) = 0 \qquad (21)$$

$$D_1B_0 + \mu_2\omega_\theta B_0 + \frac{1}{4} i\,\omega_\theta^2 B_0^2\bar{B}_0 + \frac{3}{2}\omega_\theta^3 G_2 B_0^2\bar{B}_0 + c_3 i A_0^2 \exp(-i\,\sigma_2 T_1) = 0 \qquad (22)$$

we express the complex function A_0, B_0 in the polar form as

$$A_0(T_1) = \frac{1}{2} a(T_1)\exp(i\,\gamma_1(T_1)),\ \ B_0(T_1) = \frac{1}{2} b(T_1)\exp(i\,\gamma_2(T_1)) \qquad (23)$$

where a, b, γ_1 and γ_2 are real.

Substituting equation (23) into equations (21) and (22) and separating real and imaginary part yields:

$$a' = -\mu_1 a - \frac{3}{8} a^3 G_1 + \frac{1}{4} c_2\omega_\theta^2 ab \sin\varphi_1 + (\frac{a_e}{2}\mathrm{k}_1 + \frac{3}{4}\mathrm{a}_e\mathrm{k}_3\mathrm{a}^2)\sin\varphi_2 \qquad (24)$$

$$a\gamma_1' = \frac{3}{8}\alpha_3 a^3 - \frac{1}{4} c_1 a^3 - \frac{1}{4} c_2\omega_\theta^2 ab \cos\varphi_1 - (\frac{a_e}{2}\mathrm{k}_1 + \frac{3}{4}\mathrm{a}_e\mathrm{k}_3\mathrm{a}^2)\cos\varphi_2 \qquad (25)$$

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$$b' = -\mu_2 \omega_\theta b - \frac{3}{8} G_2 \omega_\theta^3 b^3 - \frac{1}{2} c_3 a^2 \sin\varphi_1 \qquad (26)$$

$$b\gamma_2' = -\frac{1}{16}\omega_\theta^2 b^3 - \frac{1}{2} c_3 a^2 \cos\varphi_1 \qquad (27)$$

where $\varphi_1 = \gamma_2 - 2\gamma_1 + \sigma_2 T_1,\ \varphi_2 = \sigma_1 T_1 - \gamma_1$.

For the steady state solution $a' = b' = 0$, $\varphi_m' = 0;\ m = 1, 2$. Then it follows from equations (24)-(27) that the steady state solutions are given by

$$0 = -\mu_1 a - \frac{3}{8} a^3 G_1 + \frac{1}{4} c_2 \omega_\theta^2 ab \sin\varphi_1 + (\frac{a_e}{2} \mathrm{k}_1 + \frac{3}{4} \mathrm{a}_e \mathrm{k}_3 \mathrm{a}^2) \sin\varphi_2 \qquad (28)$$

$$a\sigma_1 = \frac{3}{8}\alpha_3 a^3 - \frac{1}{4} c_1 a^3 - \frac{1}{4} c_2 \omega_\theta^2 ab \cos\varphi_1 - (\frac{a_e}{2} \mathrm{k}_1 + \frac{3}{4} \mathrm{a}_e \mathrm{k}_3 \mathrm{a}^2) \cos\varphi_2 \qquad (29)$$

$$0 = -\mu_2 \omega_\theta b - \frac{3}{8} G_2 \omega_\theta^3 b^3 - \frac{1}{2} c_3 a^2 \sin\varphi_1 \qquad (30)$$

$$(2\sigma_1 - \sigma_2) b = -\frac{1}{16}\omega_\theta^2 b^3 - \frac{1}{2} c_3 a^2 \cos\varphi_1 \qquad (31)$$

From equations (28)-(31), we have the following cases:

Case 1: $a \neq 0$ and $b = 0$: in this case, the frequency response equation is given by:

$$(\frac{9}{64}\alpha_3^2 - \frac{3}{16}\alpha_3 c_1 + \frac{9}{64} G_1^2 + \frac{c_1^2}{16}) a^6 + (\frac{3}{4}\mu_1 G_1 - \frac{9}{16} a_e^2 k_3^2 + \frac{c_1}{2}\sigma_1 - \frac{3}{4}\alpha_3 \sigma_1) a^4 + (\mu_1^2 + \sigma_1^2 - \frac{3}{4} \mathrm{a}_e^2 k_1 \mathrm{k}_3) a^2 - \frac{a_e^2}{4} k_1^2 = 0 \qquad (32)$$

Case 2: $a = 0$ and $b \neq 0$: in this case, the frequency response equation is given by:

$$(\frac{9}{64} G_2^2 \omega_\theta^6 + \frac{1}{(16)^2}\omega_\theta^4) b^6 + (-\mu_2^2 \omega_\theta^2 - (2\sigma_1 - \sigma_2)^2) b^2 = 0 \qquad (33)$$

Case 3: $a \neq 0$ and $b \neq 0$: in this case, the frequency response equation are given by the following equations:

$$(\frac{9}{64}\alpha_3^2-\frac{3}{16}\alpha_3 c_1+\frac{9}{64}G_1^2+\frac{c_1^2}{16})a^6+(\frac{3}{4}\mu_1 G_1-\frac{9}{16}a_e^2 k_3^2-\frac{3}{4}\alpha_3\sigma_1$$

$$-\frac{3}{16}\mathrm{G}_1 c_2\omega_\theta^2\,\mathrm{b}\sin\varphi_1-\frac{3}{16}\alpha_3 c_2\omega_\theta^2\,\mathrm{b}\cos\varphi_1+\frac{c_1}{2}\sigma_1+\frac{1}{8}c_1c_2\omega_\theta^2\,\mathrm{b}\cos\varphi_1)a^4$$

$$+(-\frac{3}{4}\mathrm{a}_e^2\,k_1\,\mathrm{k}_3+\frac{1}{16}c_2^2\omega_\theta^4\,\mathrm{b}^2-\frac{c_2}{2}\mu_1\omega_\theta^2\,\mathrm{b}\sin\varphi_1+\frac{1}{2}c_2\omega_\theta^2\,\mathrm{b}\,\sigma_1\cos\varphi_1$$

$$+\mu_1^2+\sigma_1^2)a^2-\frac{a_e^2}{4}k_1^2=0 \qquad (34)$$

$$(\frac{9}{64}G_2^2\omega_\theta^6+\frac{1}{(16)^2}\omega_\theta^4)b^6+(\frac{3}{4}\mu_2\omega_\theta^4\,\mathrm{G}_2+\frac{1}{8}\omega_\theta^2(2\sigma_1-\sigma_2)b^4$$

$$+(\mu_2^2\omega_\theta^2+(2\sigma_1-\sigma_2)^2)b^2-\frac{1}{4}c_3^2a^4=0 \qquad (35)$$

3.1 Linear Solution

Now to the stability of the linear solution of the obtained fixed let us consider A_0 and B_0 in the forms

$$A_0(T_1)=\frac{1}{2}(\mathrm{p}_1-iq_1)\exp(i\,\delta_1 T_1),\;\; B_0(T_1)=\frac{1}{2}(\mathrm{p}_2-iq_2)\exp(i\,\delta_2 T_1) \qquad (36)$$

where $\mathrm{p}_1,\mathrm{p}_2,q_1$ and q_2 are real values and considering $\delta_1=\sigma_1,\delta_2=\sigma_2$.

Substituting equation (36) into the linear parts of equations (21), (22) and Separating real and imaginary parts, the following system of equations are obtained:

Case 1: for the solution ($a\neq 0$ and $b=0$), we get

$$p_1'+\mu_1 p_1+\sigma_1 q_1=0 \qquad (37)$$

$$q_1'-\sigma_1 p_1+\mu_1 q_1-\frac{a_e}{2}k_1=0 \qquad (38)$$

$$\begin{bmatrix}p_1'\\ q_1'\end{bmatrix}=\begin{bmatrix}-\mu_1 & -\sigma_1\\ \sigma_1 & -\mu_1\end{bmatrix}\begin{bmatrix}p_1\\ q_1\end{bmatrix}$$

The stability of the linear solution is obtained from the zero characteristic equation

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$$\begin{vmatrix} -(\lambda+\mu_1) & -\sigma_1 \\ \sigma_1 & -(\lambda+\mu_1) \end{vmatrix} = 0 \qquad (39)$$

Where $\lambda_{1,2} = -\mu_1 \pm i\,\sigma_1$

Since μ_1 is positive then the solutions are stable.

Case 2: for the solution ($a=0$ and $b\neq 0$), we get

$$p_2' + \mu_2\omega_\theta p_2 + \sigma_2 q_2 = 0 \qquad (40)$$

$$q_2' - \sigma_2 p_2 + \mu_2\omega_\theta q_2 = 0 \qquad (41)$$

$$\begin{bmatrix} p_2' \\ q_2' \end{bmatrix} = \begin{bmatrix} -\omega_\theta\mu_2 & -\sigma_2 \\ \sigma_2 & -\omega_\theta\mu_2 \end{bmatrix} \begin{bmatrix} p_2 \\ q_2 \end{bmatrix}$$

The stability of the linear solution is obtained from the zero characteristic equation

$$\begin{vmatrix} -(\lambda+\omega_\theta\mu_2) & -\sigma_2 \\ \sigma_2 & -(\lambda+\omega_\theta\mu_2) \end{vmatrix} = 0 \qquad (42)$$

Where $\lambda_{1,2} = -\omega_\theta\mu_2 \pm \sigma_2 i$

Since μ_2 is positive then the solutions are stable.

Case 3: for the solution ($\mathrm{a}\neq 0$ and $\mathrm{b}\neq 0$)we get

$$p_1' + \mu_1 p_1 + \sigma_1 q_1 = 0 \qquad (43)$$

$$q_1' - \sigma_1 p_1 + \mu_1 q_1 - \frac{a_e}{2}k_1 = 0 \qquad (44)$$

$$p_2' + \mu_2\omega_\theta p_2 + \sigma_2 q_2 = 0 \qquad (45)$$

$$q_2' - \sigma_2 p_2 + \mu_2\omega_\theta q_2 = 0 \qquad (46)$$

$$\begin{bmatrix} p_1' \\ q_1' \\ p_2' \\ q_2' \end{bmatrix} = \begin{bmatrix} -\mu_1 & -\sigma_1 & 0 & 0 \\ \sigma_1 & -\mu_1 & 0 & 0 \\ 0 & 0 & -\mu_2\omega_\theta & -\sigma_2 \\ 0 & 0 & \sigma_2 & -\mu_2\omega_\theta \end{bmatrix} \begin{bmatrix} p_1 \\ q_1 \\ p_2 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{a_e}{2}k_1 \\ 0 \\ 0 \end{bmatrix}$$

The stability of the linear solution in this case is obtained from the zero characteristic equation

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$$\begin{vmatrix} -(\lambda+\mu_1) & -\sigma_1 & 0 & 0 \\ \sigma_1 & -(\lambda+\mu_1) & 0 & 0 \\ 0 & 0 & -(\lambda+\mu_2\omega_\theta) & -\sigma_2 \\ 0 & 0 & \sigma_2 & -(\lambda+\mu_2\omega_\theta) \end{vmatrix} = 0 \qquad (47)$$

after extract we obtain that

$$\lambda^4 + r_1\lambda^3 + r_2\lambda^2 + r_3\lambda + r_4 = 0, \qquad (48)$$

where $r_1 = 2\mu_1 + 2\mu_2\omega_\theta$, $r_2 = \mu_2^2\omega_\theta^2 + \sigma_2^2 + 4\mu_1\mu_2\omega_\theta + \mu_1^2 + \sigma_1^2$,

$r_3 = 2\mu_2^2\omega_\theta^2\mu_1 + 2\mu_1\sigma_2^2 + 2\mu_1^2\mu_2\omega_\theta + 2\mu_2\omega_\theta\sigma_1^2$,

$r_4 = \mu_2^2\omega_\theta^2\mu_1^2 + \mu_1^2\sigma_2^2 + \mu_2^2\omega_\theta^2\sigma_1^2 + \sigma_1^2\sigma_2^2$

According to the Routh-Huriwitz criterion, the above linear solution is stable if the following are satisfied:

$r_1 > 0,\ r_1r_2 - r_3 > 0,\ r_3(r_1r_2 - r_3) - r_1^2 r_4 > 0,\ r_4 > 0.$

3.2 Non-Linear Solution

To determine the stability of the fixed points, one lets

$$a = a_{10} + a_{11},\ b = b_{10} + b_{11} \text{ and } \varphi_m = \varphi_{m0} + \varphi_{m1}, (m = 1, 2), \qquad (49)$$

Where a_{10}, b_{10} and φ_{m0} are the solutions of equations (28-31) and $a_{11}, b_{11}, \varphi_{m1}$ are perturbations

Which are assumed to be small compared to a_{10}, b_{10} and φ_{m0} .

Substituting equation (49) into equations (24-27), using equations (28-31) and keeping only the linear terms in $a_{11}, b_{11}, \varphi_{m1}$ we obtain:

Case 1: for the solution ($a \neq 0$ and $b = 0$), we get:

$$a_{11}' = (-\mu_1 - \frac{9}{8}G_1 a_{10}^2 + \frac{6}{4}a_e k_3 a_{10}\sin\varphi_{20})a_{11} + (\frac{3}{4}a_e k_3 a_{10}^2 \cos\varphi_{20} + \frac{a_e}{2}k_1)\varphi_{21} \qquad (50)$$

$$\varphi_{21}' = (\frac{\sigma_1}{a_{10}} + \frac{3}{4}c_1 a_{10} - \frac{9}{8}\alpha_3 a_{10} + \frac{6}{4}a_e k_3 \cos\varphi_{20})a_{11} - (\frac{3}{4}a_e k_3 a_{10}\sin\varphi_{20}$$

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$$+\frac{a_e}{2}k_1)\varphi_{21} \qquad (51)$$

The stability of a given fixed point to a disturbance proportional to $\exp(\lambda t)$ is determined by the roots of

$$\begin{bmatrix} -\mu_1-\frac{9}{8}G_1a_{10}^2+\frac{6}{4}a_e k_3\, a_{10}\sin\varphi_{20} & \frac{a_e}{2}k_1+\frac{3}{4}a_e k_3\, a_{10}^2\cos\varphi_{20} \\ \frac{\sigma_1}{a_{10}}+\frac{3}{4}c_1a_{10}-\frac{9}{8}\alpha_3a_{10}+\frac{6}{4}a_e k_3\cos\varphi_{20} & -\frac{a_e}{2}k_1-\frac{3}{4}a_e k_3\, a_{10}\sin\varphi_{20} \end{bmatrix}=0 \qquad (52)$$

Consequently, a non-trivial solution is stable if and only if the real parts of both eigenvalues of the coefficient matrix (52) are less than zero.

Case 2: for the solution ($a\neq 0, b\neq 0$) we get

$$a_{11}^{'}=(-\mu_1-\frac{9}{8}G_1a_{10}^2+\frac{c_1}{4}\omega_\theta^2\, b_{10}\sin\varphi_{10}+\frac{6}{4}a_e k_3\, a_{10}\sin\varphi_{20})a_{11} +(\frac{c_1}{4}a_{10}\omega_\theta^2\sin\varphi_{10})\, b_{11}+(\frac{c_1}{4}a_{10}\, b_{10}\,\omega_\theta^2\cos\varphi_{10})\varphi_{11}+(\frac{a_e}{2}k_1 +\frac{3}{4}a_e k_3\, a_{10}^2\cos\varphi_{20})\varphi_{21} \qquad (53)$$

$$\varphi_{21}^{'}=(\frac{\sigma_1}{a_{10}}+\frac{3}{4}c_1a_{10}-\frac{9}{8}\alpha_3a_{10}+\frac{c_2}{4a_{10}}\omega_\theta^2\, b_{10}\cos\varphi_{10}+\frac{6}{4}a_e k_3\cos\varphi_{20})a_{11} +(\frac{c_2}{4}\omega_\theta^2\cos\varphi_{10})\, b_{11}-(\frac{c_2}{4}\omega_\theta^2\, b_{10}\sin\varphi_{10})\varphi_{11}-(\frac{3}{4}a_e k_3\, a_{10}\sin\varphi_{20} +\frac{a_e}{2}k_1)\varphi_{21} \qquad (54)$$

$$b_{11}^{'}=(-c_3a_{10}\sin\varphi_{10})a_{11}-(\mu_2\omega_\theta+\frac{9}{8}\omega_\theta^3\, b_{10}^2G_2)\, b_{11}-(\frac{1}{2}c_3a_{10}^2\cos\varphi_{10})\varphi_{11} \qquad (55)$$

$$\varphi_{11}^{'}=-(\frac{c_3a_{10}}{b_{10}}\cos\varphi_{10}+\frac{2\sigma_1}{a_{10}}-\frac{9}{4}\alpha_3a_{10}+\frac{3c_1}{2}a_{10}+\frac{c_2}{2a_{10}}\omega_\theta^2\, b_{10}\cos\varphi_{10} +3a_e k_3\cos\varphi_{20})\, a_{11}+(\frac{\sigma_2-2\sigma_1}{b_{10}}-\frac{3}{16}b_{10}\omega_\theta^2-\frac{c_2}{2}\omega_\theta^2\cos\varphi_{10})\, b_{11}$$

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$$+(\frac{c_3}{2b_{10}}a_{10}^2\sin\varphi_{10}+\frac{c_2}{2}\omega_\theta^2\,\mathrm{b}_{10}\sin\varphi_{10})\varphi_{11}+(\frac{a_e}{a_{10}}k_1\sin\varphi_{20}$$
$$+\frac{3}{2}a_e k_3 a_{10}\sin\varphi_{20})\varphi_{21} \qquad (56)$$

The stability of a particular fixed point with respect to perturbations proportional to $\exp(\lambda t)$ depends on the real parts of the roots of the matrix. Thus, a fixed point given by equations (53)-(56) is asymptotically stable if and only if the real parts of all roots of the matrix are negative.

4. Numerical Results

The behavior of the given system of equations (1), (2) has been solved numerically applying Runge-Kutta 4th order method [8, 9] and frequency response equations via Maple 16. Fig. 1 illustrates the response and phase-plane for the non-resonant system at some practical values of the equations parameters.

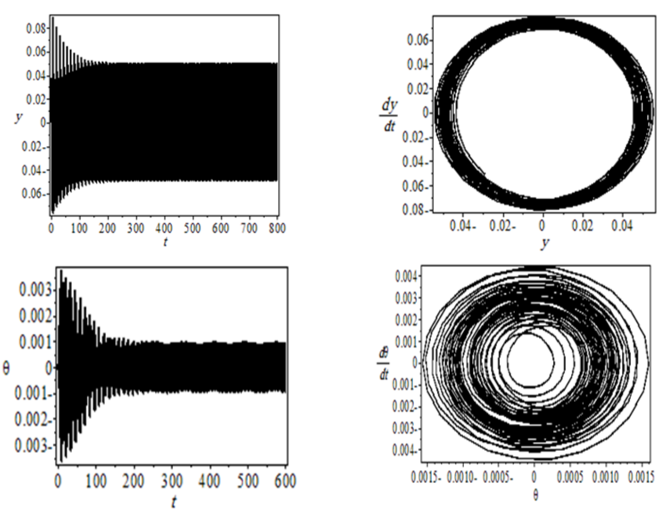

Fig. 1: The basic case of the system without controller.

$\mu_1=0.0155, a_e=0.1212,\ \alpha_2=-0.0432638, \alpha_3=0.512554, \omega_\theta=1.5$

$k_1=0.507566, k_2=-0.442311, k_3=0.2576$

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From this fig., we can see that the system is stable with the steady state amplitude y and are 0.05 and 0.001 respectively, and the phase plane shows the system is stable with multi limit cycles.

4.1 Resonance Cases

Some of the deduced resonance cases of the plant without the control are studied numerically as shown in Table 1. From this table, we see that the amplitude increasing at the resonance cases and the worst case is the simultaneous resonance case when ω_{Θ}-2= 0, Ω-1= 0, which the amplitudes are increased to about 800% compared with the basic case shown in fig. 1. It can be shown that the amplitudes y and Θ are increased to 0.4 and 0.2 respectively compared with the system without control shown in Fig.2 , which means that the system needs to reduced the amplitude of vibration as controlled, in Fig. 3.

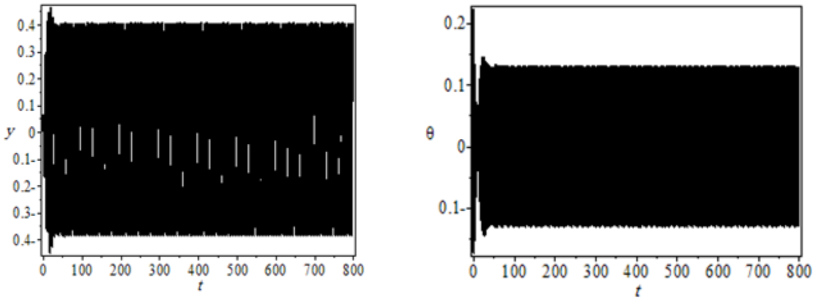

Fig. 2: System behavior without controller

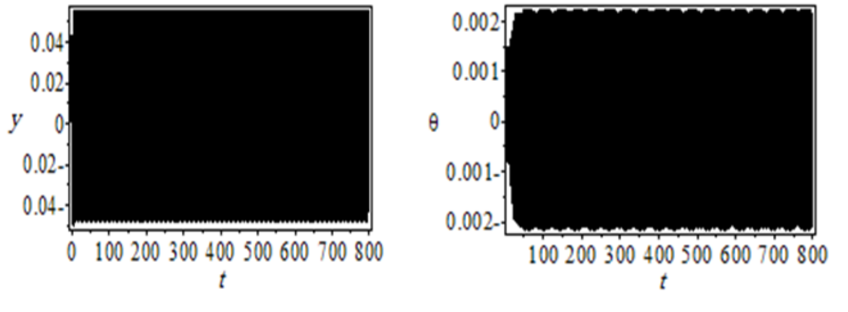

Fig. 3: System behavior with controller

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4.2 Effect of the Controller

Fig. 3, illustrates the results when the controller is effective, when ω_Θ-2= 0 and Ω-1= 0. The effectiveness of the controller is Ea (Ea =steady state amplitude of the main system without controller / steady state amplitude of the main system with controller) are about 10 and 63.64.

Table 1. Resonance Cases

Resonance cases	Y without control	Y with control	E_a	*Θ* without control	*Θ* with control	E_a
ω_Θ +Ω= 1	0.1	0.05	2	0.01	0.004	2.5
ω_Θ -Ω= 1	0.1	0.04	2.5	0.01	0.005	2
ω_Θ -Ω= -1	0.015	0.014	1.07	0.00005	0.00004	1.25
ω_Θ= Ω	0.05	0.049	1.02	0.0005	0.0004	1.25
ω_Θ +Ω= 2	0.12	0.09	1.33	0.01	0.0035	2.86
ω_Θ - Ω= 2	0.05	0.05	1	0.001	0.0007	1.43
ω_Θ =Ω- 2	0.0095	0.009	1.06	0.00001	0.00001	1
ω_Θ-2=0, Ω-1=0	0.4	0.04	10	0.14	0.0022	63.64

4.3 Effect of Parameters

The amplitude of the y is monotonic decreasing function of the damping coefficient μ_1 as shown in Fig. 4a. But the amplitude of the system is monotonic increasing function of the non-linear coefficients a_e, k_1 and k_3 as shown in Figs. 4b, 4c and 4d. But the amplitude of the system is monotonic decreasing function of the gain coefficient G_1 as shown in Fig. 4e.

The amplitude of the *Θ* is monotonic decreasing function of the damping coefficient μ_2 and gain coefficient G_2 as shown in figs. 4f, 4g.

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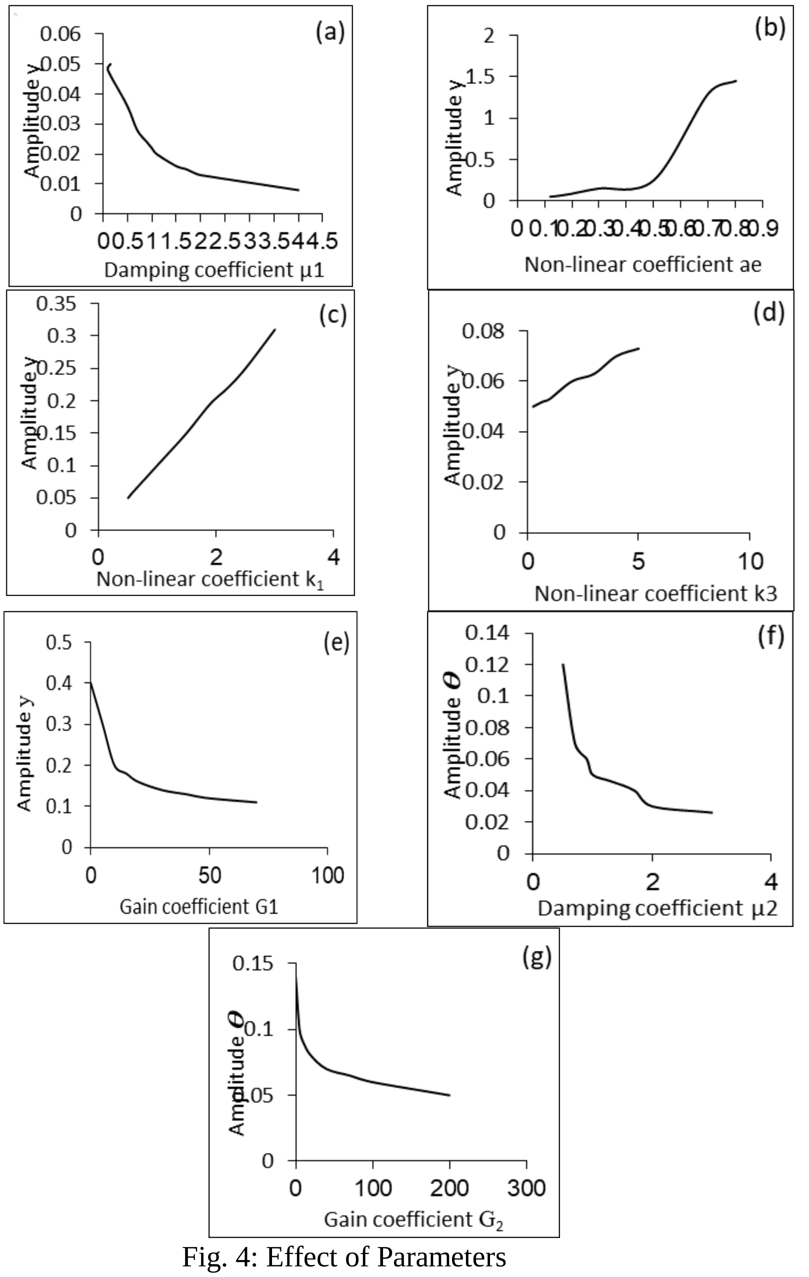

Fig. 4: Effect of Parameters

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4.5 Response Curves

The frequency response equations (32), (33),(34) and (35) are nonlinear algebraic equations of a, b. These equations are solved numerically as shown in Figs. 5-8. From case 1 where $a \neq 0, b = 0$: Fig .5 shows that the steady state amplitudes of the system are monotonic decreasing functions in μ_1, G_1 and monotonic increasing functions in k_1, a_e.

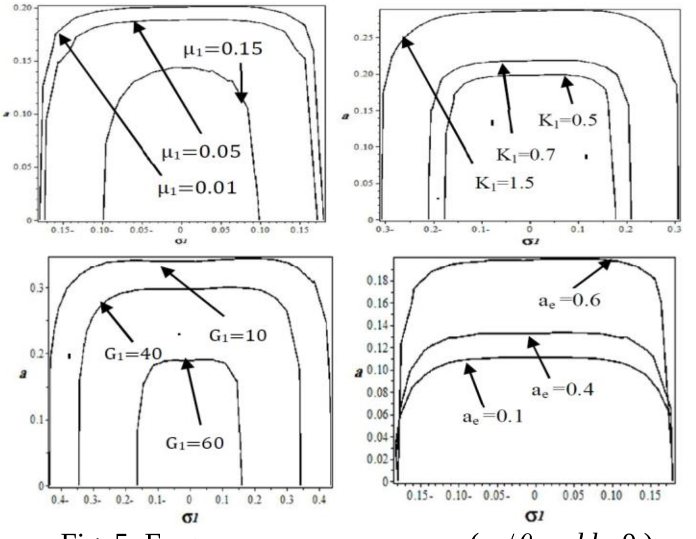

Fig. 5: Frequency response curves (*a≠0 and b*=0)

From case 2, where $a = 0, b \neq 0$: Fig. 6, shows that the steady state amplitudes of the system are monotonic decreasing functions in ω_θ, G_2 and monotonic increasing functions in μ_2.

From case 3, where $a \neq 0, b \neq 0$: Fig. 7, shows that the steady state amplitudes of the system are monotonic decreasing functions in μ_1, G_1 and monotonic increasing functions in k_1, a_e.

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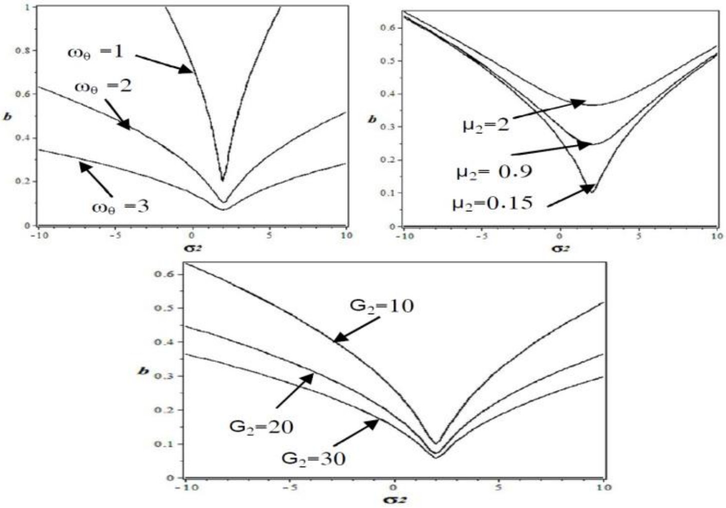

Fig. 6: Frequency response curves (*a=0 and b*≠0)

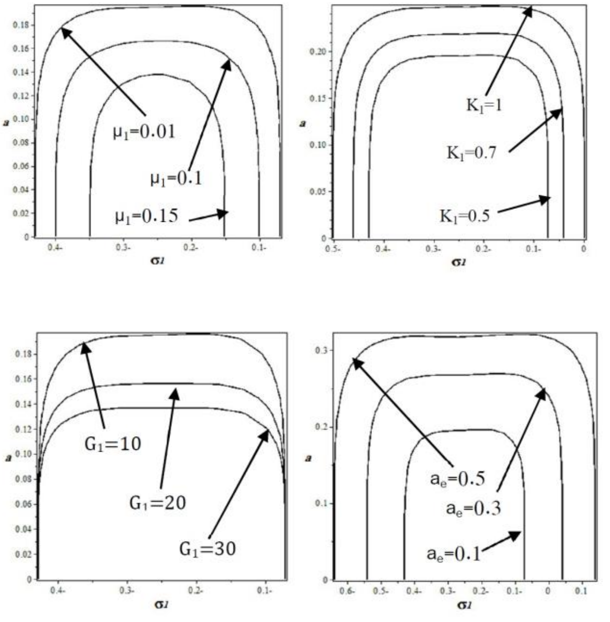

Fig. 7: Response curves (*a*≠0 *and b* ≠0)

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Fig. 8, shows that the steady state amplitudes of the system are monotonic decreasing functions in $\omega_\theta, \mu_2, G_2$ and monotonic increasing functions in c_3.

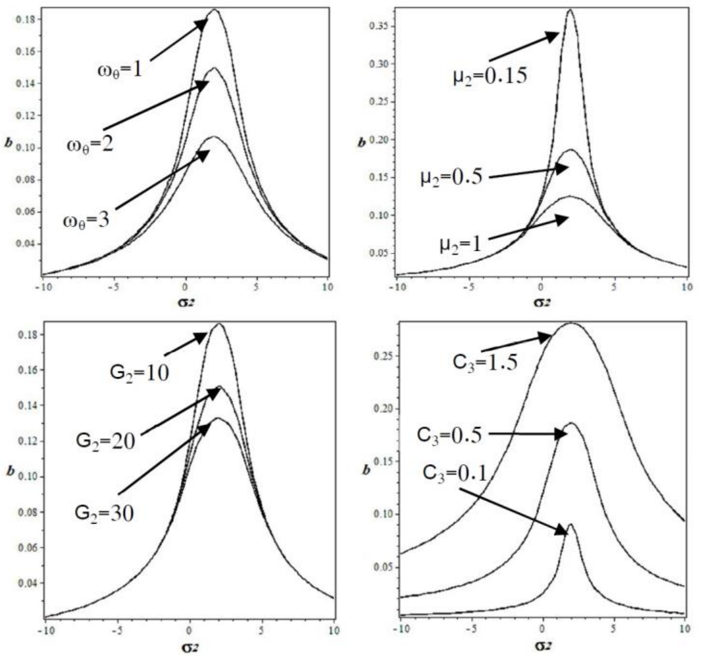

Fig. 8: Response curves ($a \neq 0$ *and* $b \neq 0$)

5. Conclusions

From the above study the results may be concluded

1. The worst resonance case is the simultaneous resonance case $\omega_\theta - 2 \cong 0$, $\Omega - 1 \cong 0$, the amplitudes are increased to about 800% compared with the basic case.

2. The control can reduced the amplitudes y and θ to about 0.052 and 0.002 respectively compared with the system without control.

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3. The amplitude of the y is monotonic decreasing functions to the damping coefficient μ_1, G_1, But it is monotonic increasing function of the nonlinear coefficient k_1, a_e .

4. The amplitude of the θ is monotonic decreasing function of the ω_θ, μ_2, G_2. But it is monotonic increasing function of the c_3.
5. The system stability near the resonance case applying the frequency-response equations.
6. In the linear solutions if μ_1, μ_2 is positive then the solutions are stable.

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